always use more primitive or fundamental mathematical concepts with more general mathematical methods.

One may ask why some theories of sets and elementary algebra are specially classified as fundamental tools for all mathematical arguments. Of course, what is called fundamental or elementary may also vary as our knowledge or common sense changes. Therefore, some of today's theories of sets and algebra might be replaced by more desirable ones in the future. In this sense, perhaps we cannot obtain what is ultimately fundamental or elementary. Even so, I am convinced that the *linguistic feature* of mathematics itself will never change in our development of knowledge. With the theory of sets, algebraic theory provides a structure to describe our words, sentences, and even our logic used in mathematics or ordinary mathematical arguments.

As long as our knowledge is represented by language, it can be *coded* into algebraic or at least elementary set-theoretical objects. Therefore, the set-theoretic and algebraic methods used in this book must provide a basic framework for arguments that depend not on well-behaved (continuous and/or differentiable) mappings, but on the well-founded minimum requirements for mappings on a primitive finite set of points. They provide a possible framework for *coding* themselves as knowledge to be used in well-founded theories constructed by themselves.

Homology theory is the algebraic study of the connectivity characteristics of a space. Čech-type theory begins this study by approximating the space by sufficiently refined open coverings, thus reducing the connectivity problem to the intersection property among open sets. In Figure 3a, 1-dimensional space X is covered by open covering $\mathscr{M} = \{M_0, M_1, M_2, M_3\}$. In this case, X is approximated by the set of abstract points and lines represented in Figure 3b. Each abstract point (vertex or 0-dimensional simplex) is associated with the name of an open set in the covering, and each line (1-dimensional simplex) indicates that two open sets related to the two vertices of the line intersect. The totality of such abstract simplices (the abstract *complex*) is called the *nerve of covering* $\mathscr{M}$. By taking refinement $\mathscr{N} = \{N_0, N_1, N_2, N_3, N_4\}$ of covering $\mathscr{M}$ (Figure 4a), we obtain the nerve of covering $\mathscr{N}$ as a better approximation for space X (Figure 4b).

A careful reader might think that even if a covering refinement gives a better approximation for the connectivity of space, it may also cause a problem: The dimensions of approximating simplices become too high. In Figure 5a, the nerve of the covering, two open sets, offers a sufficiently good approximation for space X. If we take further refinement for the

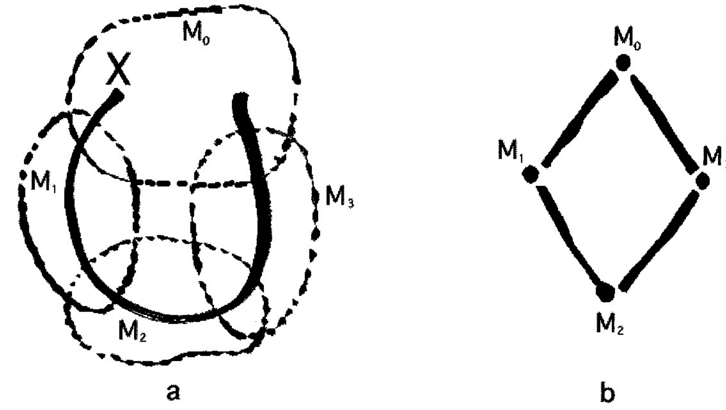

Figure 3: Nerve of covering I

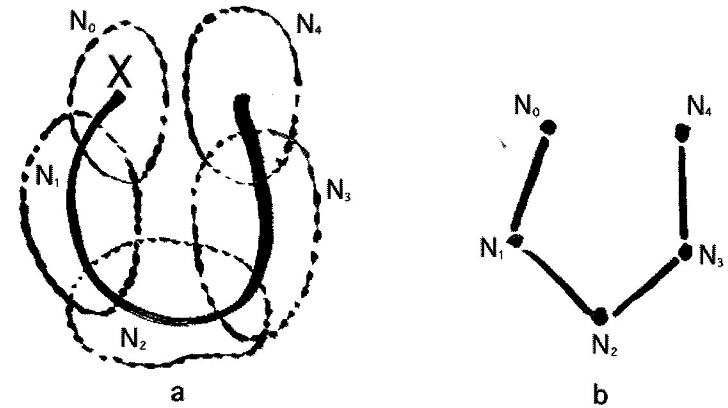

Figure 4: Nerve of covering II

covering, as shown in Figure 5b, the dimensions of simplices approximating X increase, which apparently cannot be reduced under any process of taking refinements. How can we argue that 5b is a better approximation than 5a?

The answer precisely illustrates the homological argument. In homology theory, the difference between the shapes in Figures 5a and 5b is not important. Both sets are called *acyclic*, which is essentially identified with a *single point* under homological arguments. Homology theory associates topological space X with set $H_q(X)$, (q-th homology group of X) with an algebraic structure (e.g., groups, modules, and vector spaces) for each dimension $q = 0, 1, 2, \ldots$. Intuitively, the q-th homology group, $H_q(X)$,